Exercise 31

Explain, using Theorems 4, 5, 7, and 9, why the function is continuous at every number in its domain. State the domain.

$$M(x) = \sqrt{1 + \frac{1}{x}}$$

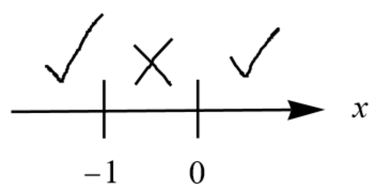
Solution

Both 1 and $\frac{1}{x}$ are continuous functions at all numbers in their respective domains by Theorem 7, and their sum $1 + \frac{1}{x}$ is also a continuous function for $x \neq 0$ by Theorem 4. $M(x)$ is a composition of the square root function and this sum, so by Theorem 9 this is also a continuous function at all numbers in its domain. Find the domain of $M(x)$ by requiring what's under the square root to be a nonnegative number.

$$1 + \frac{1}{x} \geq 0$$

$$\frac{x+1}{x} \geq 0$$

The critical points are $x = -1$ and $x = 0$. Partition the number line at these points, and test whether the inequality is true or false in each interval.



As a result,

$$x \leq -1 \quad \text{or} \quad x \geq 0.$$

Combining these two conditions with $x \neq 0$, the domain of $M(x)$ is

$$(-\infty, -1] \cup (0, \infty).$$